DISTRIBUTED GENERATION: AN ALTERNATIVE TO ELECTRIC UTILITY INVESTMENTS IN SYSTEM CAPACITY

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Abstract

Distributed generation technologies offer electric utilities an alternative to large system capacity investments. This paper presents a simplified method to determine the value of deferring electric utility capacity investments using distributed generation. Consideration is given to both economic and technical factors, including uncertainty in the price of distributed generation. The technical evaluation is based on measured data from a 500 kW distributed generation PV plant in Kerman, California. Results indicate that the cost savings associated with deferring capacity investments can be accurately estimated using only seven economic parameters and one day's worth of distributed generation output and utility system data.

Key Words

distributed generation; electric utility investment; and uncertainty

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Electric utilities have historically satisfied customer demand by generating electricity centrally and distributing it through an extensive transmission and distribution network. As demand increases, the utility generates more electricity. Once demand increases beyond a certain level, however, the capacity of the generation, transmission, and distribution systems can become constrained. The traditional utility response to these constraints is to build new facilities (bold portion of Figure 1).

An alternative approach under consideration by utilities is to satisfy demand locally and incrementally by investing in distributed generation.[1] Distributed generation facilities are strategically sited to deliver electricity where it is needed as shown in Figure 2. This can relieve capacity constraints on the generation, transmission, and distribution systems and obviate the need to build new facilities.

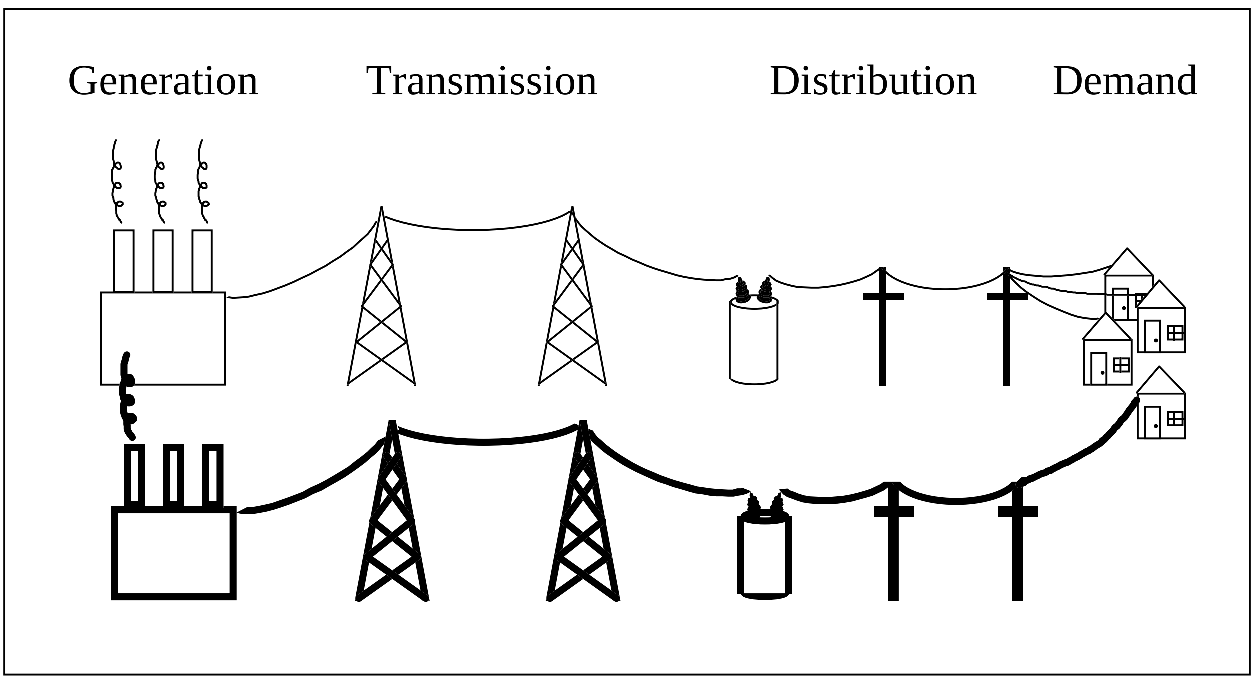

Figure 1. Traditional utility response to demand increases is to build new facilities.

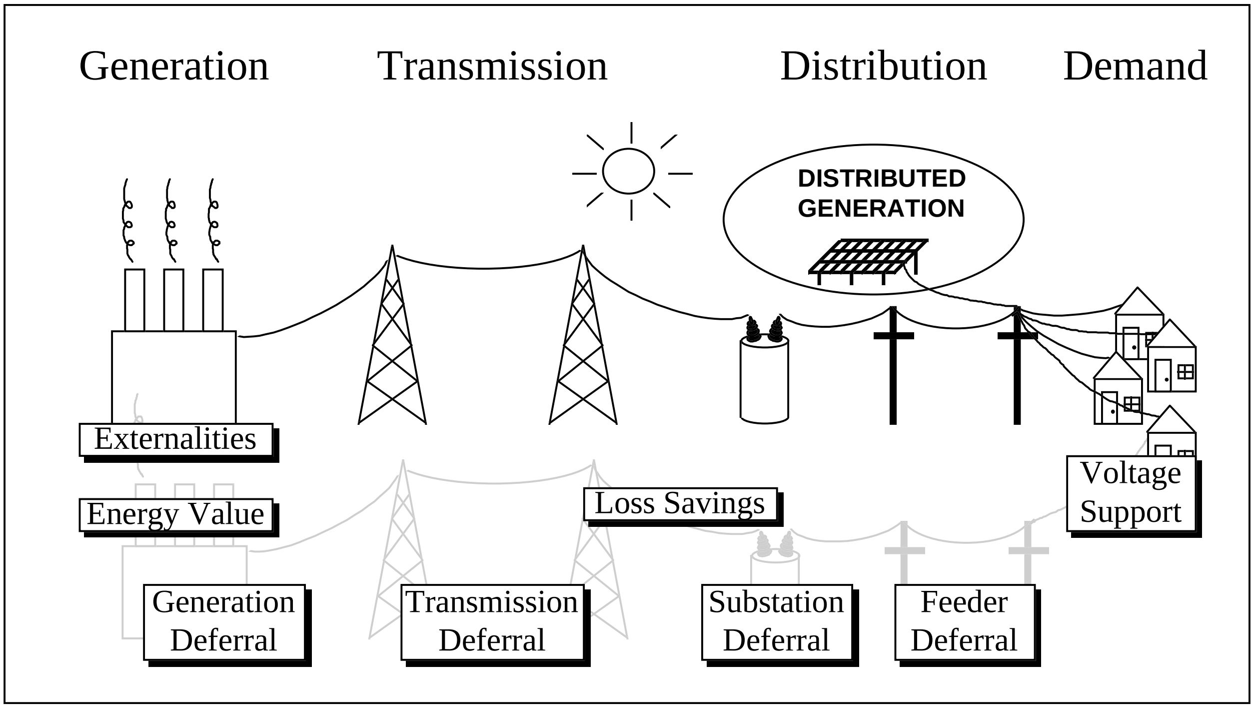

Figure 2. The value of distributed generation to the utility system.

Evaluating the cost-effectiveness of distributed generation is a crucial resource planning issue. Many have assessed cost-effectiveness by dividing the utility system into many parts and estimating distributed generation's value to each part. When this is done, total value can be composed of ten or more individual components[2] including distribution feeder,[3] substation transformer,[4] transmission system, generation system, voltage support,[5] reliability, energy savings, electrical loss savings,[6] minimum load savings, externalities, modularity and flexibility,[7] and financial risk reduction values.[8]

Although impressive, this list of value components suggests that determining the value of distributed generation requires a team of experts assembled from each department within the utility. This paper describes a simplified evaluation process based on the observation that distributed generation is of value because it reduces variable costs or defers capacity investments (see Figure 3). This suggests that two methods repeatedly applied throughout the utility system are sufficient to determine the value of distributed generation: one to determine the reduction in variable costs and the other to determine the value of deferring capacity investments. These two methods are repeated for each portion of the utility system to determine total value.

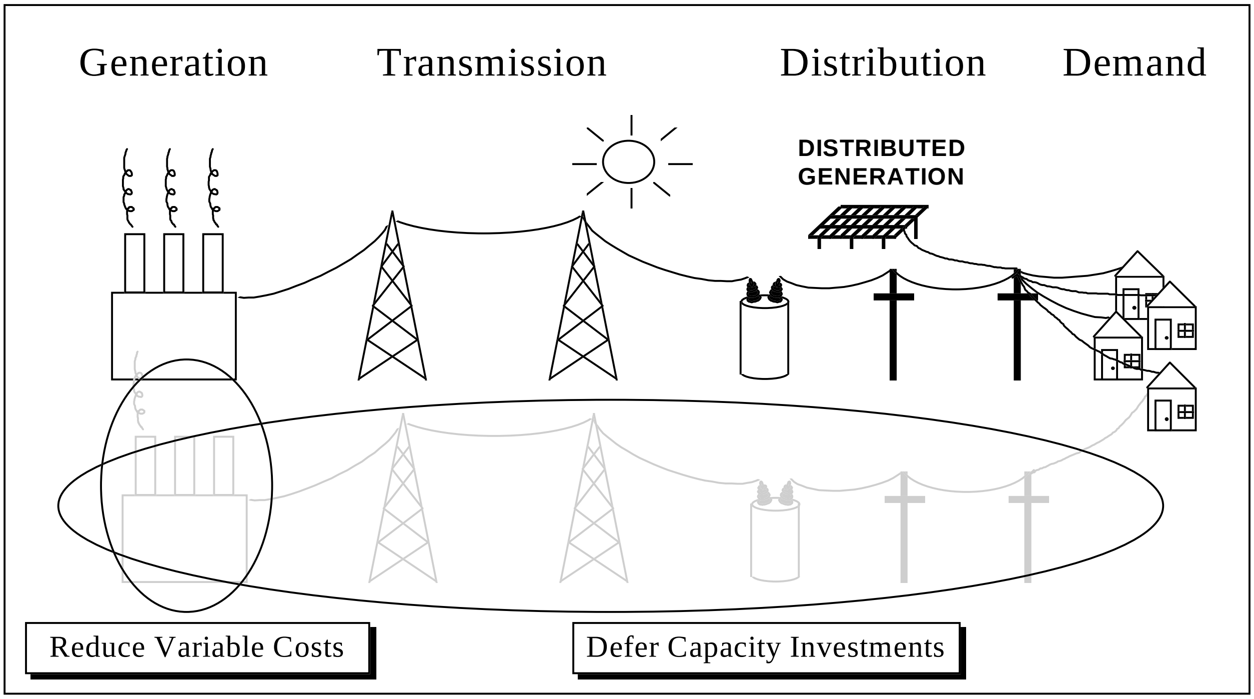

Figure 3. Distributed generation reduces variable costs and defers capacity investments.

Objective

This paper describes a method to determine the value of deferring capacity investments in terms of the break-even distributed generation price. The break-even price is the price at which the utility is economically indifferent between making the traditional investment (i.e., a system upgrade) or investing in distributed generation. It is calculated by performing an economic evaluation[9] for a distributed generation resource with perfect operating characteristics and then adjusting the result based on a technical evaluation[10] that considers the specific characteristics of the generation technology and its application. The break-even price for each part of the utility system needs to be calculated and the results summed to obtain the total break-even price. A distributed generation investment is economical if the actual distributed generation price is less than this total break-even price.

The first section of this paper presents the economic evaluation for an ideal distributed generation resource that defers a capacity investment. It determines the break-even price for a fully dispatchable distributed generation resource. The second section presents the technical evaluation to adjust for actual distributed generation technology type, location, and utility system characteristics. Data from a 500 kW distributed generation photovoltaic (PV) plant in Kerman, California, are used to illustrate the technical evaluation. This plant is the first utility demonstration plant designed and built to measure the benefits of distributed generation. Conclusions are presented in the final section.

Economic Evaluation

This section develops the economic evaluation to determine the break-even distributed generation price to defer planned capacity investments to upgrade a specific part of the utility system. It begins by performing a present value analysis under certainty. It then demonstrates how to incorporate uncertainty in the price of distributed generation. The section concludes with a sample calculation.

Fully Utilize Capacity

Large investments have large capacities. In some cases, such as the generation system, capacity may be fully utilized immediately upon investment. In other cases, such as in parts of the transmission and distribution system, there may be unused capacity for a period of years. This situation is illustrated by the darkened portions of Figure 4. The figure shows that an investment with a capacity of *C* is made every *T* years.[11] Thus, there is excess system capacity immediately after the investment is made. Distributed generation capacity, in comparison, is installed frequently in very small sizes. This results in a situation in which capacity and demand are always equal. This eliminates the unused capacity portions of Figure 4.

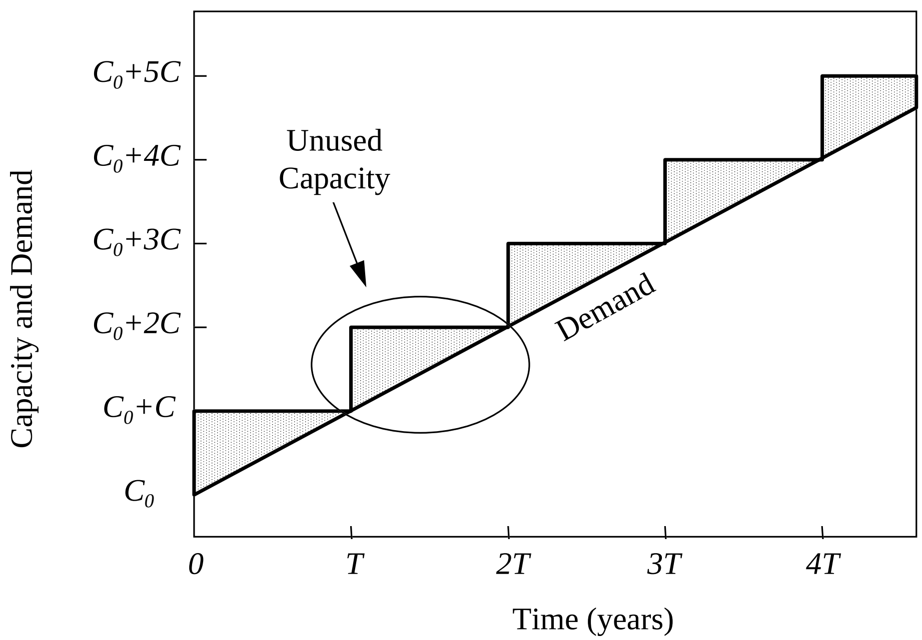

Figure 4. Distributed generation can reduce unused capacity.

Orans,[12] along the lines of Turvey, [13] developed a method to estimate the value of deferring traditional utility investments within an electric distribution system based on demand-side management programs. In essence, he calculates the added value of fully utilizing system capacity. He obtains a capacity expansion plan based on existing demand, calculates its cost, and then repeats this with a slightly altered demand. The per unit value, or break-even price, is the amount of money that can be spent to defer a system investment.

Orans' approach is taken in this research with two modifications. As presented in Figure 5, system capacity is slightly increased by adding distributed generation rather than reducing demand. More significantly, the capacity expansion plan is estimated rather than fully specified. The plan is estimated by segmenting the utility system into areas according to load transfer capability, classifying homogeneous capacity expenditures (e.g., feeders are one category, transformers are a second category, etc.), and assuming linear demand growth and that a similar piece of equipment is installed after its capacity is exhausted. Figure 5 presents the original (dashed line) and deferred (solid line) capacity expansion plans. The markings on the axis correspond to the timing and capacity of the deferred plan. The difference between the two plans is that, at time equal to 0, a small amount of distributed generation is installed. This increases the capacity of the system by C_{DG} and defers the original plan by T_{DG} years.

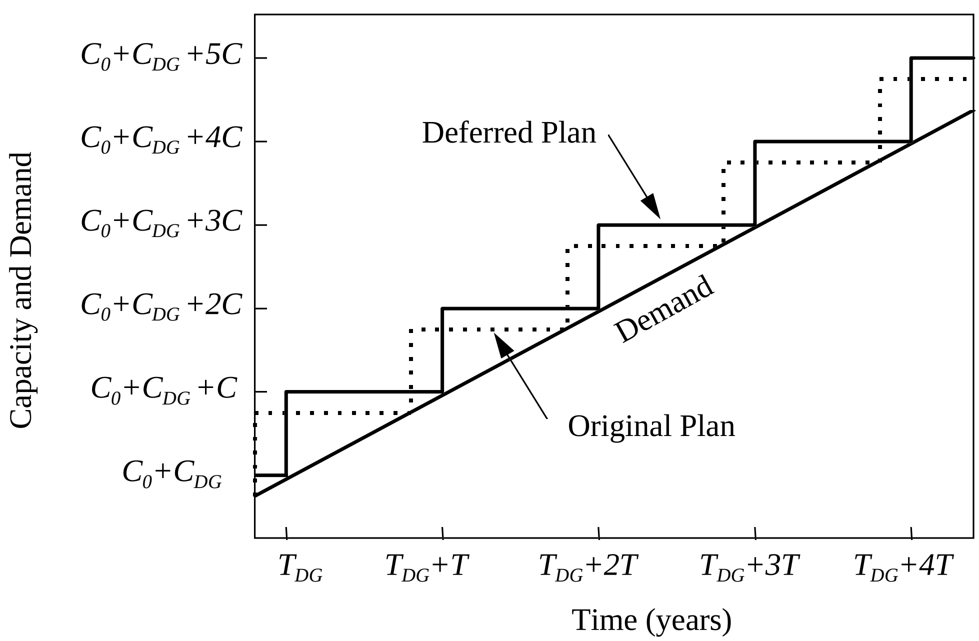

Figure 5. Break-even price is calculated by altering the original capacity expansion plan.

The lifetime cost of one homogeneous portion of the original plan is the sum of the cost of each piece of equipment discounted according to when the expenditure is made. In order for the utility to be indifferent between the traditional investment or the distributed generation investment, economic equilibrium requires that the present value[14] of the original plan must equal the present value of the altered plan plus the distributed generation cost.

$$\underbrace{\sum_{k=0}^{\infty} I \exp^{-(\rho-\alpha)T*k}}_{\substack{\text{Present Value of} \\ \text{Original Plan}}} = \underbrace{\sum_{k=0}^{\infty} I \exp^{-(\rho-\alpha)(T*k+T_{DG})} + P*C_{DG}}_{\substack{\text{Present Value of} \\ \text{Deferred Plan}}} \qquad (1)$$

where I is the full cost of the system investment, P is the break-even distributed generation price for this portion of the system, T is the years between investments,[15] $T*k$ is the year in which the $k+1$ investment is made, T_{DG} is the years of deferral associated with the distributed generation investment,[16] C_{DG} is the capacity of the distributed generation investment, and $\rho-\alpha$ is the real discount rate, which is assumed to be positive, relative to the system investment (i.e., it is the discount rate, or cost of capital, minus the investment escalation rate).

Equation (1) is solved by reducing the series on both sides of the equation and assuming that the distributed generation is installed incrementally to satisfy increased demand.[17] The solution is that the break-even price for this portion of the system equals the average cost of the system upgrade adjusted for the full capacity utilization of the distributed generation investment.

$$P = \overbrace{\left[\frac{I}{C}\right]}^{\substack{\text{Average}\\ \underline{\text{Cost}}}} \quad \overbrace{\left[\frac{(\rho-\alpha)T}{1-\exp^{-(\rho-\alpha)T}}\right]}^{\substack{\text{Full Capacity}\\ \underline{\text{Utilization}}}} . \tag{2}$$

A case study was performed by Pacific Gas and Electric Company (PG&E) to assess the distributed benefits of a 500 kW PV plant located near Kerman, California.[18] That study offers an opportunity to validate Equation (2). The study determined that one benefit was the deferral of a transformer upgrade. It calculated a break-even distributed generation price for the substation transformer deferral of $780/kW. It assumed that a new 16 MW, $1.4 Million transformer was needed, there was 0.1 MW/year of load growth, and the real discount rate was 6 percent.

The load growth and transformer capacity assumptions imply that the number of years between investments (*T*) is 160 years (16 MW divided by 0.1 MW/year). Inputting this and the other assumptions into Equation (2) yields a break-even distributed generation price to the substation transformer of about $840/kW. This compares well with the detailed case study result.

Expected Distributed Generation Cost Reductions

While Equation (2) accounts for full distributed generation capacity utilization, it does not account for expected distributed generation cost reductions. Suppose distributed generation is not cost-effective when a capacity investment is required but will soon become cost-effective. As Figure 6 suggests, a large capacity investment at time 0 precludes the need for additional capacity for *T* years (i.e., the number of years until additional capacity is needed as shown in

Figure 4) and thus, the opportunity to obtain capacity value from distributed generation. This requires adjusting the break-even distributed generation price in anticipation of expected cost reductions.

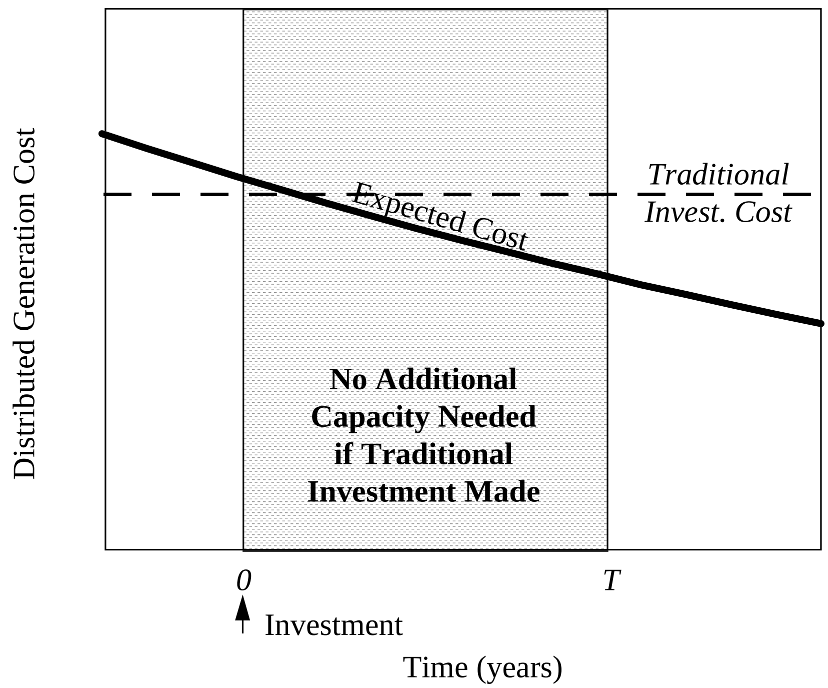

Figure 6. Expected distributed generation cost reductions can benefit the utility.

When there are expected distributed generation cost reductions relative to the original investment, these reductions are accounted for by replacing the first piece of equipment in the original capacity expansion plan with an incremental installation (at the same rate as the demand growth) of distributed generation. Since the plans are identical after *T* years, the break-even distributed generation price is the price at which the total expenditure on distributed generation over *T* years equals the cost of the first piece of equipment in the expansion plan (*I*). That is,

$$\int_0^T P\delta \exp^{-(\rho-\alpha_{DG})t}\, dt = I \qquad (3)$$

where α_{DG} is the escalation rate in the price of distributed generation and δ is the annual rate of growth in demand (this equals the capacity of the investment, *C*, divided by the number of years between investments).

The solution to Equation (3), multiplied by an identity and rearranged, is

$$P = \overbrace{\left[\frac{I}{C}\right]}^{\substack{\text{Average}\\ \underline{\text{Cost}}}} \overbrace{\left[\frac{(\rho-\alpha)T}{1-\exp^{-(\rho-\alpha)T}}\right]}^{\substack{\text{Full Capacity}\\ \underline{\text{Utilization}}}} \overbrace{\left[\left(\frac{\rho-\alpha_{DG}}{\rho-\alpha}\right)\left(\frac{1-\exp^{-(\rho-\alpha)T}}{1-\exp^{-(\rho-\alpha_{DG})T}}\right)\right]}^{\substack{\text{Expected Dist. Gen.}\\ \underline{\text{Cost Reductions}}}}. \qquad (4)$$

A comparison of Equations (4) to (2) reveals that the break-even price is identical in both cases except for the addition of a term that adjusts for the difference between the escalation rates of the price of distributed generation and the original investment.

Distributed Generation Cost Uncertainty

The preceding subsections assumed that the future is known with certainty. In reality, there are many uncertainties associated with the future.[19] One of those uncertainties is the price of distributed generation. Figure 7 presents a decision maker's knowledge about the cost of a distributed generation technology with declining, uncertain costs. The middle line represents the expected cost and the dashed lines represent two possible cost paths; many paths are possible.

The decision maker must decide between a large capacity investment or incrementally installing distributed generation when system capacity is constrained. If the decision maker selects the large capacity investment in year 0, Figure 7 suggests that it will be *T* years until any additional capacity is required. Making the investment is a good decision from an expected cost perspective since the cost of distributed generation is higher than the cost of the traditional investment until almost year *T*. The decision maker cannot take advantage of low costs, however, if distributed generation costs decline rapidly as in the case with the Possible Cost 1 path.

This suggests that there is value to deferring the system upgrade decision in order to resolve uncertainty about the future cost of distributed generation. Figure 8 illustrates that the value is a revised long-term expected cost estimate of satisfying demand using distributed generation. This new information is obtained by deferring the upgrade decision for T_{DG} years at the short-term cost of one increment of distributed generation with a capacity of C_{DG}. This positions the decision maker to cost-effectively satisfy long-term demand using distributed generation after T_{DG} years if costs decline more rapidly than expected (Revised Expected Cost 1) while keeping the option open of a traditional system upgrade if costs increase (Revised Expected Cost 2).

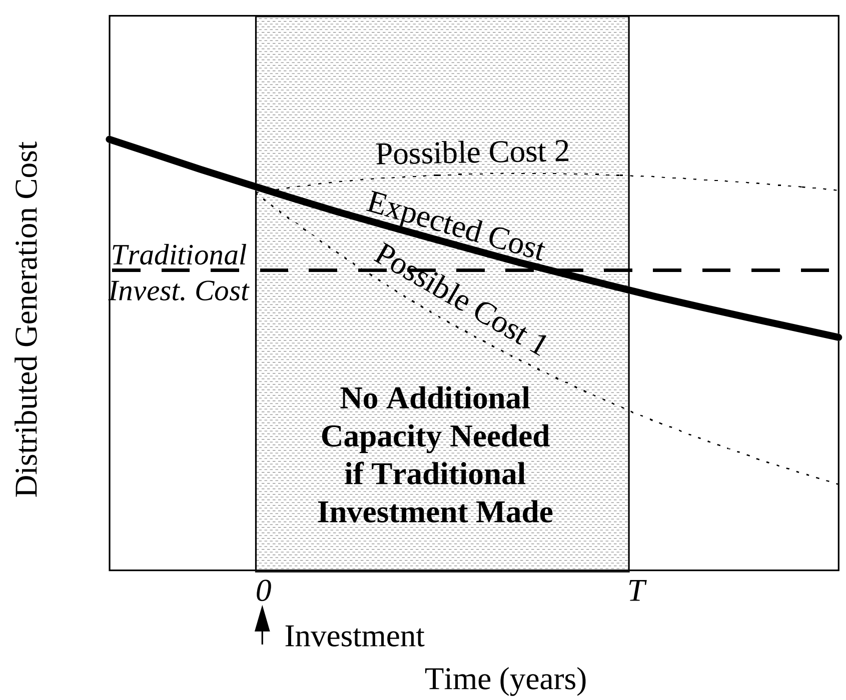

Figure 7. Large capacity investments eliminate additional capacity needs.

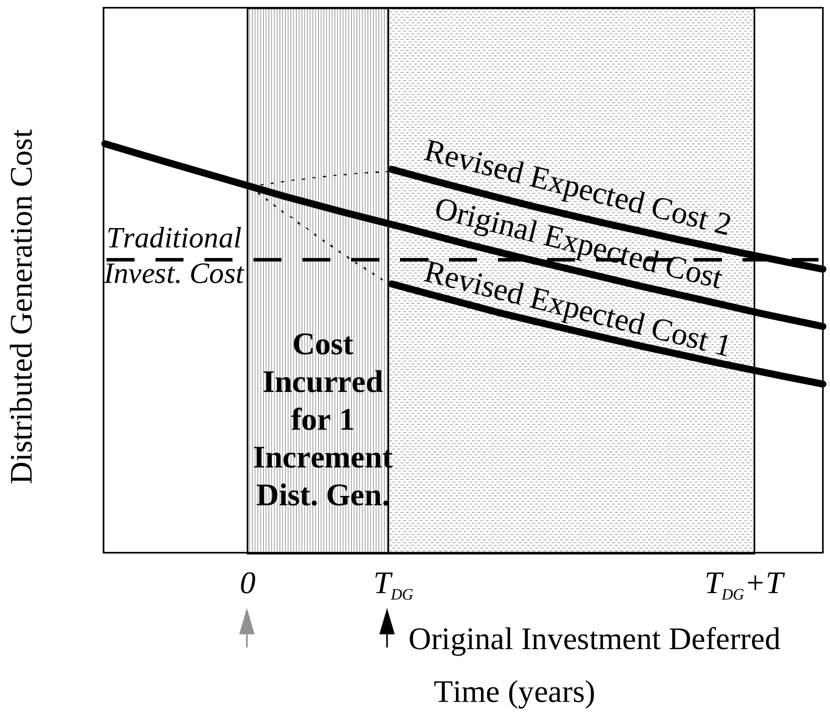

Figure 8. Distributed generation cost uncertainty can benefit the utility.

In order to determine what this value is, first consider the case when there is no uncertainty. The solid lines in Figure 9 represent the expected cost of two alternatives in providing system capacity for T years (i.e., the length of time that the traditional system investment provides adequate capacity to the utility). Both are plotted versus the current distributed generation price. The expected distributed generation cost is the present value of the distributed generation expenditures made incrementally over time. The cost of this alternative increases with distributed generation price. The expected system investment cost is simply the system investment cost because it is incurred immediately. The figure indicates that distributed generation is the least-cost alternative for distributed generation prices less than $P_{No\ Option}$ while a system investment is the least-cost alternative for prices greater than $P_{No\ Option}$. Thus, the break-even price when there is no uncertainty is $P_{No\ Option}$.

Now consider the case when there is uncertainty. The lower dashed line in Figure 9 represents the option value of having an alternative available. The option is defined so that installing distributed generation incrementally is the base case and making the system investment is the decision. Thus, the option value is the value of having the option available to make the traditional system investment rather than being forced to satisfy demand incrementally with distributed generation regardless of price. The option value curve starts out at zero because there is very little value to having the system investment alternative available if the price of distributed generation is near zero. This is because the utility has no incentive to consider other alternatives when the distributed generation alternative is so inexpensive. The option value increases with distributed generation price because the value of having another alternative available becomes more desirable as the cost of the distributed generation alternative increases.

Thus, when there is uncertainty, there are two costs associated with making the traditional system investment. First, there is the cost of the investment itself. Second, there is the cost of the lost option of being able to make the system investment at some point in the future. Thus, the traditional system investment should be made at the point where the investment cost (I) plus the option value (i.e., the cost of the lost option) equals the cost of the distributed generation alternative. This corresponds to a break-even price of P_{Option}.

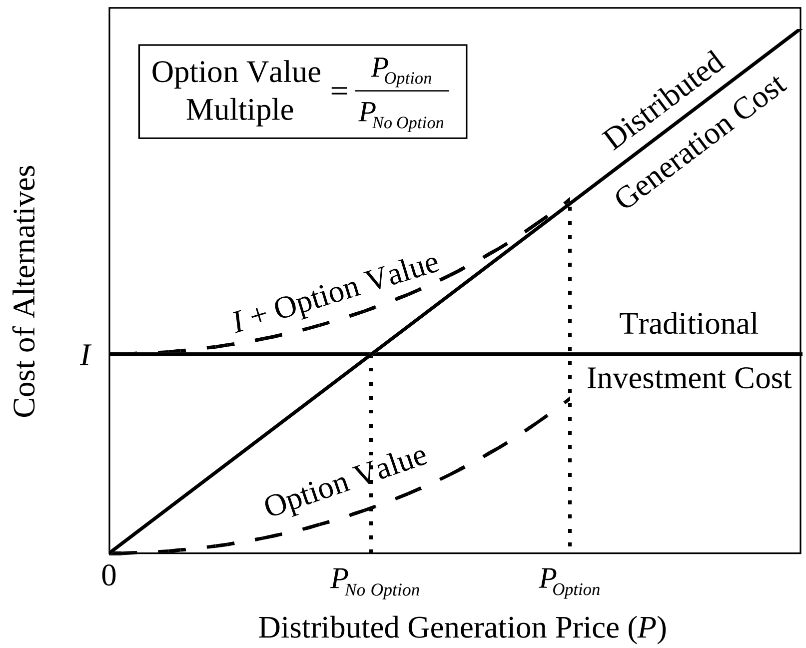

Figure 9. Break-even distributed generation price under uncertainty.

An explicit solution to this problem is obtained by performing a contingent claims[20] or dynamic programming analysis. A contingent claims analysis is applicable if the uncertainty is spanned by existing assets[21] while dynamic programming is used if it is not. The approaches differ in that the appropriate discount rate is given in the contingent claims analysis while the discount rate must be specified in the dynamic programming analysis; since distributed generation is not a publicly traded asset, a dynamic programming analysis is performed. The solution to this problem is stated here and derived in detail in the Appendix.

$$P = \overbrace{\left[\frac{I}{C}\right]}^{\substack{\text{Average}\\ \underline{\text{Cost}}}} \overbrace{\left[\frac{(\rho-\alpha)T}{1-\exp^{-(\rho-\alpha)T}}\right]}^{\substack{\text{Full Capacity}\\ \underline{\text{Utilization}}}} \overbrace{\left[\left(\frac{\rho-\alpha_{DG}}{\rho-\alpha}\right)\left(\frac{1-\exp^{-(\rho-\alpha)T}}{1-\exp^{-(\rho-\alpha_{DG})T}}\right)\right]}^{\substack{\text{Expected Dist. Gen.}\\ \underline{\text{Cost Reductions}}}} \overbrace{\left[\frac{\beta}{\beta-1}\right]}^{\substack{\text{Option Value}\\ \underline{\text{Multiple}}}} \qquad (5)$$

where β is defined in the Appendix.

A comparison of Equations (5) to (4) reveals that the break-even price is identical in both cases except for the addition of the option value multiple. The option value multiple, which is always greater than one, is the ratio of P_{Option} to $P_{No\ Option}$ in Figure 9. It reflects the decision making flexibility associated with the distributed generation investment alternative over the traditional system upgrade investment.

Break-Even Price for Ideal Resources

Equation (5)'s simplicity makes it very useful in determining the break-even price of distributed generation. In particular, only seven variables are required to perform the evaluation. This subsection illustrates how to use the equation. In doing so, it demonstrates that a key variable in determining the break-even price is how often the traditional system investment needs to be made.

Consider two capacity constraint situations: one which requires a generation system investment every year and the other a transmission system investment every 30 years. Assume that both the generation and transmission system investments are made in 50 MW sizes, cost $50 Million, and are escalating at a rate of 5 percent. The discount rate is 10 percent, there is no expected distributed generation cost escalation, and there is 20 percent distributed generation cost uncertainty.

Figure 10 presents the break-even price for progressively grouped terms in Equation (5). The bottom area corresponds to average cost, the second to fully utilized capacity,[22] the third to distributed generation cost reductions, and the fourth to distributed generation cost uncertainty. The area labeled Cost Uncertainty would disappear if there was no distributed generation cost uncertainty and the area labeled Expected Cost Reductions would disappear if the system investment and the distributed generation investment had the same cost escalation rate.

The most important thing to notice in Figure 10 is that the break-even price is very sensitive to the number of years between investments (*T*). The intuition behind this is that lumpiness of the system investment is fully described from an economic perspective by *T*. Consider several examples of how this figure could be used. The break-even distributed generation price is about $1.00/Watt when deferring generation investments that need to be made annually. The break-even distributed generation price is about $4.00/Watt when deferring transmission system investments that are made every 30 years. If the distributed generation defers both generation and transmission system investments, the break-even distributed generation price is the sum of the two, or about $5.00/Watt.

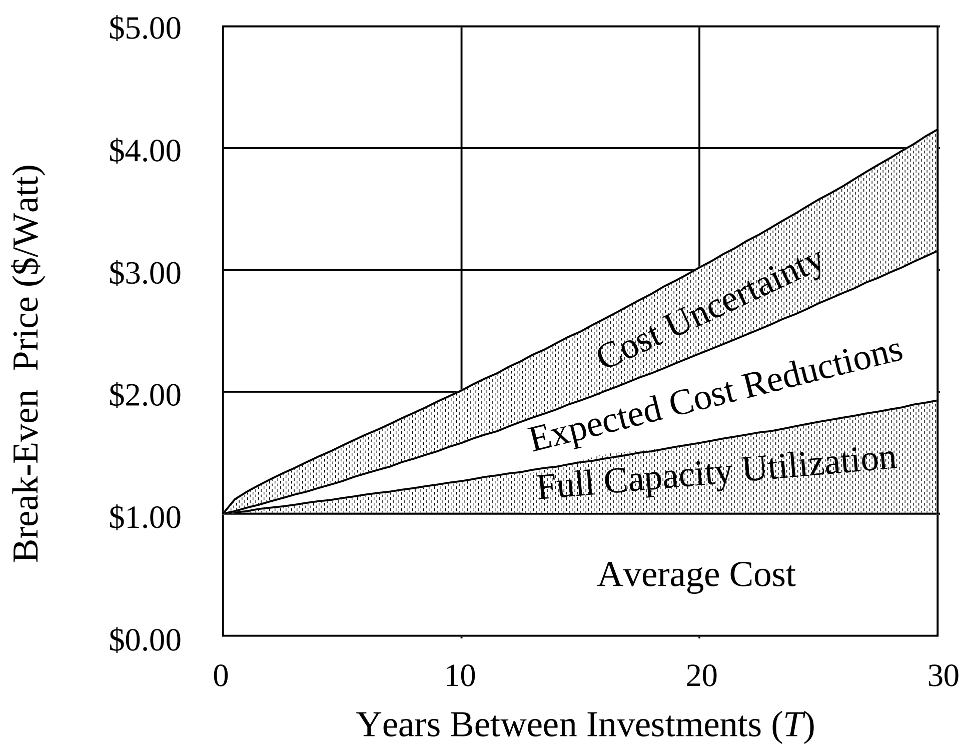

Figure 10. Break-even price increases with the number of years between investments.

Technical Evaluation

The break-even price derived in the previous section is for an ideal distributed generation resource. Although adequate in performing a "back of the envelope" calculation, greater accuracy is obtained by reflecting the actual increase in system capacity based on estimated distributed generation output, distributed generation location, and interaction with the utility system. This section draws from research performed using a year's worth of measured plant and utility data from the 500 kW PV plant in Kerman, California.[23] It begins with an illustration of how to obtain performance data for a PV plant, demonstrates how to adjust the output based on location, shows how to assess the interaction between distributed generation output and the utility system using a transmission system example, and then concludes by combining the technical result with the economic result from the previous section.

Distributed Generation Output

Distributed generation output is required to evaluate the increase in system capacity. It simply equals plant rating for fully dispatchable plants with no emissions or operational restrictions (e.g., a generator in an area with no pollution restrictions); plants with limitations (e.g., non-dispatchable solar power plants or battery plants with operational restrictions), however, require further analysis. This section describes how to estimate output for a PV plant.

Weather is a crucial factor in determining PV plant output due to its non-dispatchability. Likewise, weather is a crucial factor in determining peak loads (e.g., summer peaking utilities experience peak loads on hot days). Thus, it is important to estimate PV plant output using weather conditions that coincide with system capacity constraints.

This is accomplished by simulating PV plant output using measured weather data corresponding to the measured load data. Output simulated as described by Wenger and Gordon[24] has proven to be highly accurate when compared to measured PV plant output data.[25]

Change in System Losses

Reducing system load by generating power locally has the multiplicative effect of reducing system losses. This arises from the engineering principle that losses decrease with load. Distributed generation output needs to be adjusted to reflect this.

Loss savings are based on the investment's location relative to the distributed generation plant. They include the change in losses on all equipment between the investment to be deferred and the distributed generation facility. The example in this section is for a transmission system deferral. Loss savings on the transmission system relative to distributed generation located on a feeder includes the change in losses on the feeder, substation, and transmission system.

A first order approximation of loss savings is the system average. The next level of detail is obtained by evaluating feeder loss savings because it is at that location that the distributed generation resource can change the load profile (and thus the corresponding losses) the most.[26] Feeder loss savings can be estimated using either a utility planning model or a simple model.[27] Distributed generation plant output adjusted for loss savings on the peak load day in 1993 at Kerman, California, is presented in Figure 11.

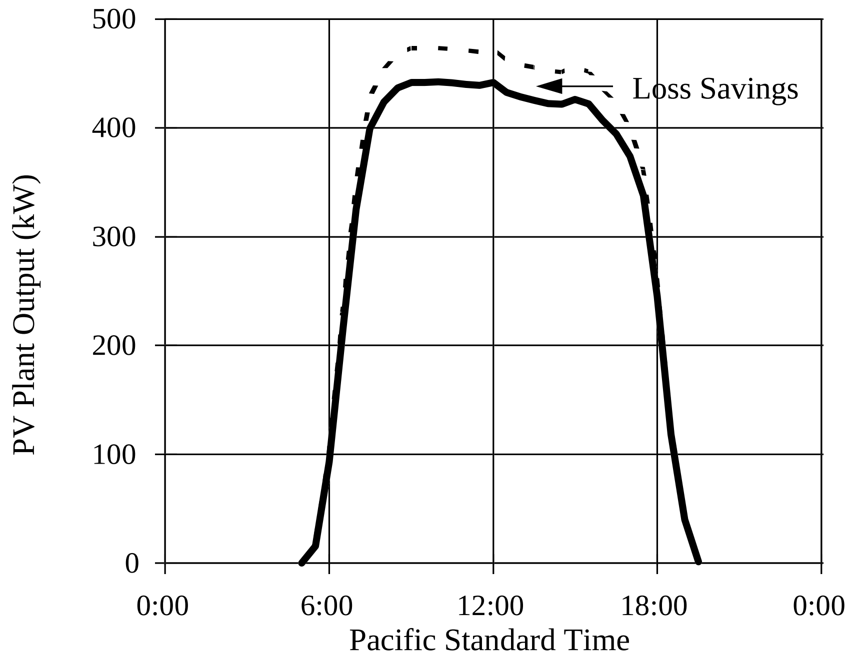

Figure 11. PV plant output adjusted for system losses.

Interaction with the Utility System

The increase in system capacity is determined by combining plant output (adjusted for the change in system losses) with utility system characteristics, including load data and system capacity. Load data can be measured while capacity must be estimated. System capacity is assumed to be constant if a peak load reduction approach is taken. A more accurate approach is to determine capacity based on weather conditions and accepted engineering standards.[28]

Increased system capacity is the difference between peak capacity with and without the resource. Although the analysis can be performed using a large data set (e.g., one year), research[29] suggests that examining the worst day of the year may suffice if plant output patterns are relatively constant (such as is the case for PV). In the case of the 500 kW Kerman PV plant, identical results are obtained when the data set contains either a year's worth of data or the worst day of the year. The worst day of the year is defined to be a combination of high load and low plant output.

Figure 12 presents weather-adjusted capacity, measured load, and measured PV plant output[30] for the 1993 peak day for Pacific Gas and Electric Company's sub-transmission system near the Kerman PV plant. The figure suggests that the need for capacity is highly correlated with PV plant output, with the PV plant increasing system capacity by 90 percent of the plant's rating.[31]

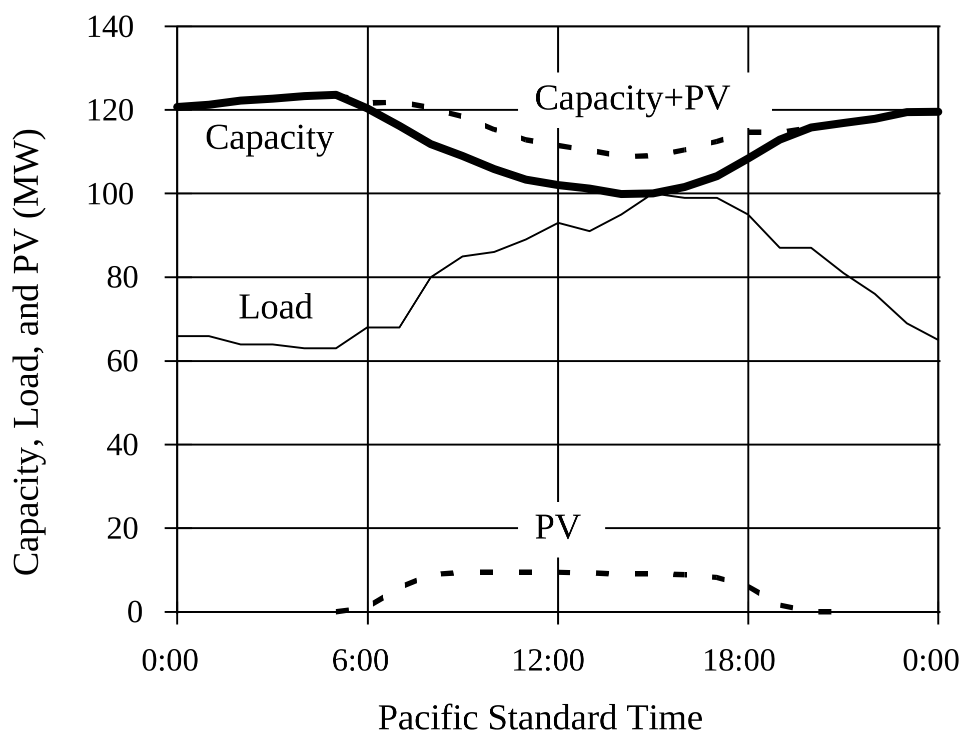

Figure 12. PV output correlates well with the need for system capacity.

Break-Even Price for Actual Resource

The break-even price for an actual resource in a specific location equals the break-even price of an ideal resource (calculated in the previous section) times the actual resource's technical effectiveness. Suppose that the break-even distributed generation price for the transmission system is the result presented in the previous section. Since the technical evaluation indicates that the PV plant is 90 percent effective in increasing transmission system capacity, the break-even price for an actual resource in this example is $4.00/Watt (the result from the previous section) times 90 percent, or $3.60/Watt. One would repeat this analysis for all portions of the utility system and sum the results to determine the total break-even price.

Conclusions

This research has demonstrated that determining the value of distributed generation can be simplified by categorizing the value of distributed generation as coming from either reducing variable costs or deferring capacity investments. This suggests that two methods repeatedly applied throughout the utility system are sufficient to determine the value of distributed generation: one to determine the reduction in variable costs and the other to determine the value of deferring capacity investments.

This paper presented a method to calculate the break-even distributed generation price for deferring capacity investments, giving consideration to economic and technical issues. Break-even price is calculated by adjusting the large investment's average capacity cost for the economic effects of elimination of excess system capacity, expected distributed generation cost reductions, and distributed generation cost uncertainty, and the technical effect of utility system interaction based on plant output, plant location, and utility system characteristics.

One of the most important results from this research is the small amount of data required to perform the evaluation: only seven parameters are required to perform the economic evaluation and one day's worth of distributed generation output and utility system data can be used to perform the technical evaluation. That makes this approach simple to apply and thus, may have wide applicability.

The next steps of this research are to develop a simplified method to estimate variable cost savings. Once this is completed, the results can be used to determine the total break-even price of distributed generation in various applications.

Appendix

This appendix uses dynamic programming to formulate and solve the problem of how distributed generation price uncertainty affects the break-even price.[32] It is assumed that the price of distributed generation follows geometric Brownian motion. This means that the change in the price of distributed generation, dP, equals

$$dP = \alpha_{DG} \, Pdt + \sigma_{DG} \, Pdz \tag{6}$$

where α_{DG} is the expected price escalation, σ_{DG} is the standard deviation in this estimate, and dz is the increment of a Wiener process. Assuming that distributed generation price follows geometric Brownian motion implies that the expected price at time t equals $Pe^{\alpha_{DG} t}$. This corresponds to the standard economic assumption of exponential price escalation.

Demand is satisfied incrementally using distributed generation so that the decision maker's option is when to make the large investment. The value associated with the large investment is the cost savings of not having to take the distributed generation approach.

Let $V(P)$ denote the large investment value with that of all future capacity addition options as capacity is exceeded. Cost savings over the next short interval of time associated with the large investment is the break-even distributed generation price for this part of the system times load growth ($P\delta\, dt$). Demand will exceed system capacity with a probability $\lambda\, dt$ during this short interval of time, where λ is 0 if demand does not exceed capacity or else it is a very large number. The utility will return to installing distributed generation if the generation cost is low enough when demand exceeds capacity and it will once again hold the option, worth $F(P)$, of making another system upgrade some time in the future.

$$V(P) = P\delta\ dt + (1-\lambda\ dt)\mathrm{E}\left[V(P+dP)\exp^{-\rho\ dt}\right] + \lambda\ dt\mathrm{E}\left[F(P+dP)\exp^{-\rho\ dt}\right] . \tag{7}$$

Expanding Equation (7) using Ito's Lemma (with the recognition that value is a function of both the price of distributed generation and time) and simplifying results in a partial differential equation.[33]

$$\tfrac{1}{2}\sigma_{DG}^{2} P^2 V_{PP} + (\alpha_{DG} - \alpha) P V_P + V_t - (\rho - \alpha + \lambda) V + P\delta + \lambda F = 0 . \tag{8}$$

The complete (homogeneous and particular) solution to Equation (8) is

$$V(P) = \left[AP^{\beta} + \frac{P\delta}{\rho - \alpha_{DG} + \lambda}\right]\left[1 - \exp^{(\rho - \alpha_{DG} + \lambda)(t-T)}\right] + F(P) \tag{9}$$

where β is the positive solution to the quadratic equation

$$\tfrac{1}{2}\sigma_{DG}^{2}\beta(\beta - 1) + (\alpha_{DG} - \alpha)\beta - (\rho - \alpha + \lambda) - \left[\frac{\rho - \alpha_{DG} + \lambda}{\exp^{(\rho - \alpha_{DG} + \lambda)(T-t)} - 1}\right] = 0 \tag{10}$$

and the option value, $F(P)$, equals $AP^{\beta'}$.[34]

Unknown constants are determined using the value-matching and smooth-pasting conditions.[35] The value-matching condition is that the option value equals the investment value minus the investment cost [i.e., $F(P) = V(P) - I$] and the smooth-pasting condition is the derivative of this with respect to P [i.e., $F_P(P) = V_P(P)$].

The value-matching condition is

$$F(P) = \left[AP^{\beta} + \frac{P\delta}{\rho - \alpha_{DG} + \lambda} \right] \left[1 - \exp^{(\rho - \alpha_{DG} + \lambda)(t - T)} \right] + F(P) - I \tag{11}$$

which simplifies to

$$AP^{\beta} + \frac{P\delta}{\rho - \alpha_{DG} + \lambda} = \frac{I}{1 - \exp^{(\rho - \alpha_{DG} + \lambda)(t - T)}} \ . \tag{12}$$

The value-matching and smooth-pasting results are combined to arrive at a break-even price, *P*, devoted to this part of the system.

$$P = \left[\frac{\rho - \alpha_{DG} + \lambda}{\delta} \right] \left[\frac{I}{1 - \exp^{(\rho - \alpha_{DG} + \lambda)(t - T)}} \right] \left[\frac{\beta}{\beta - 1} \right] \ . \tag{13}$$

Equation (13) is the same as Equation (5) in the body of the text if the investment has not yet been made (so *t* equals zero) and the time between investments is greater than zero (so lambda equals zero).

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[1] Daniel S. Shugar, "Photovoltaics in the Utility Distribution System: The Evaluation of System and Distributed Benefits," *Proceedings of the 21st IEEE PV Specialists Conference*, Kissimmee, Florida, May 1990.

[2] Rick Lambeth and Tom Lepley, "Distributed Photovoltaic System Evaluation by Arizona Public Service Company," *Proceedings of the 23rd IEEE PV Specialists Conference*, Louisville, Kentucky, May 1993.

[3] Dean L. Travers and Daniel S. Shugar, "The Value of Grid-Support Photovoltaics to Electric Distribution Lines," forthcoming in *Progress in Photovoltaics: Research and Applications* in 1995.

[4] Mohammed M. El-Gassier, Ken P. Alteneder, and John Bigger, "Enhancing Transformer Dynamic Rating Through Grid Application of Photovoltaic Arrays," *Proceedings of the 23rd IEEE PV Specialists Conference*, Louisville, Kentucky, May 1993.

[5] Tom Hoff, Howard J. Wenger, and Brian K. Farmer, "The Value of Grid-Support Photovoltaics in Providing Distribution System Voltage Support," *Proceedings of the 1994 American Solar Energy Society Annual Conference*, San Jose, California, June 1994.

[6] Tom Hoff and Daniel S. Shugar, "The Value of Grid-Support Photovoltaics in Reducing Distribution System Losses," forthcoming in *IEEE Transactions on Energy Conversion* in 1995.

[7] Peter A. Morris, Karen L. Wong, and Robert O. Wood, "Value of Flexibility & Modularity of Distributed Generation Technologies: Photovoltaics at PG&E's Kerman Substation," EPRI Investment Strategies Project, Palo Alto, California, September 1993.

[8] Shimon Awerbuch, "Distributed Generation Revisited: A Market-Based Approach to Valuing the Kerman Station Upgrade," Internal Report Prepared for Niagara Mohawk Power Corporation, Syracuse, New York, March 1994.

[9] See Thomas E. Hoff, *Cost-Minimizing Incremental Investments Under Uncertainty*. Engineering-Economic Systems Department Dissertation, Stanford University, forthcoming in 1995, for a further discussion of the economic evaluation, particularly as it relates to uncertainty.

[10] See Thomas E. Hoff and Howard J. Wenger, "The Value of Photovoltaics in the Distribution System: The Kerman Grid-Support Project," Pacific Gas and Electric Company, report forthcoming, in 1995 for a more detailed discussion about the technical evaluation.

[11] C_0 is the capacity of the system at time 0 before any investment is made.

[12] Ren Orans, *Area-Specific Marginal Costing for Electric Utilities: A Case Study of Transmission and Distribution Costs*, Civil Engineering Department Dissertation, Stanford University, September 1989.

[13] Ralph Turvey, *Optimal Pricing and Investment in Electric Supply*, Cambridge, Massachusetts: MIT Press, 1968.

[14] See Richard A. Brealey and Stewart C. Myers, *Principles of Corporate Finance*, fourth edition, New York: McGraw-Hill, 1991, for a discussion of present value.

[15] T equals the capacity of the investment, C, divided by load growth, δ. This is true when there is only one equipment rating in a given category (e.g., all transformers installed in an area are 20 MW transformers). The

problem requires a slightly different formulation if there are multiple equipment ratings within a homogeneous category.

[16] T_{DG} equals the capacity of the distributed generation investment, C_{DG}, divided by load growth, δ.

[17] $\sum_{k=0}^{\infty} I \exp^{-(\rho-\alpha)T*k} = \frac{I}{1-\exp^{-(\rho-\alpha)T}}$ and $\sum_{k=0}^{\infty} I \exp^{-(\rho-\alpha)(T*k+T_{DG})} = \frac{I \exp^{-(\rho-\alpha)T_{DG}}}{1-\exp^{-(\rho-\alpha)T}}$. Thus, Equation (1) can be rewritten as $\frac{I\left(1-\exp^{-(\rho-\alpha)T_{DG}}\right)}{1-\exp^{-(\rho-\alpha)T}} = P*C_{DG}$. When T_{DG} is small, the numerator on the right hand side can be approximated so that $\frac{I(\rho-\alpha)T_{DG}}{1-\exp^{-(\rho-\alpha)T}} = P*C_{DG}$. The substitution that $\frac{T_{DG}}{C_{DG}} = \frac{T}{C}$ is used to yield the final result.

[18] Daniel Shugar, et. al., "Benefits of Distributed Generation in PG&E's Transmission and Distribution System: A Case Study of Photovoltaics Serving Kerman Substation," Pacific Gas and Electric Company Report 007.5-92.9, November 1992.

[19] See Kevin A. Hassett and Gilbert E. Metcalf, "Energy Conservation Investment: Do Consumers Discount the Future Correctly?" *Energy Policy* Vol 21, No 6, June 1993, pp 710-716, for a discussion of how this applies to consumer investments in energy conservation.

[20] This is also called a real options analysis.

[21] In essence, this means that the risk in the price of distributed generation is traded in the market.

[22] The areas are additive. This means that the second area is built on top of the first area.

[23] Thomas E. Hoff and Howard J. Wenger, "The Value of Photovoltaics in the Distribution System: The Kerman Grid-Support Project," Pacific Gas and Electric Company, report forthcoming, in 1995.

[24] Howard J. Wenger and J. M. Gordon, "PVGRID: A User-Friendly PC Software Package for Predicting Utility-Intertie Photovoltaic System Performance," *ISES Solar World Congress*, Denver, Colorado, August 1991.

[25] Howard J. Wenger, Thomas E. Hoff, and Brian K. Farmer, "Measuring the Value of Distributed Photovoltaic Generation: Final Results of the Kerman Grid-Support Project," *First World Conference on Photovoltaic Energy Conversion*, Waikaloa, Hawaii, December 1994.

[26] For example, the 500 kW PV plant at Kerman represents 20 percent of feeder capacity where the plant is located while it represents less than 0.003 percent of the generation system capacity.

[27] Tom Hoff and Daniel S. Shugar, "The Value of Grid-Support Photovoltaics in Reducing Distribution System Losses," forthcoming in *IEEE Transactions on Energy Conversion* in 1995.

[28] Standards such as ANSI/IEEE C57.92-1981, *Guide for Loading Mineral-Oil-Immersed Power Transformers Up to and Including 100 MVA with 55 °C or 65 °C Winding Rise* for transformers and IEEE Standard 738-1993, *IEEE Standard for Calculating the Current-Temperature Relationship of Bare Overhead Conductors* for conductors can be used.

[29] Thomas E. Hoff and Howard J. Wenger, "The Value of Photovoltaics in the Distribution System: The Kerman Grid-Support Project," Pacific Gas and Electric Company, report forthcoming, in 1995.

[30] PV plant output is measured at the 500 kW PV plant at Kerman, adjusted up by 7 percent for loss savings, and then multiplied by 20 to represent a 10 MW PV plant.

[31] Analysis of the Kerman project suggests that the weather-adjusted capacity approach can result in substantially greater allowable distributed generation penetrations than a constant capacity. Results indicate that a PV plant with output patterns similar to the actual 500 kW Kerman PV plant would be 90 percent effective at increasing transmission system capacity for PV plant sizes up to 15 MW if transmission system capacity is weather-adjusted versus 3 MW if capacity is not weather adjusted.

[32] The reader is urged to consult a reference such Avinash K. Dixit and Robert S. Pindyck, *Investment Under Uncertainty*, New Jersey: Princeton University Press, 1994 for further details.

[33] The discount and distributed generation escalation rates (ρ and α_{DG}) are relative to the large investment escalation rate (α).

[34] The option value is determined in a manner similar to the value except there is no return in holding the option over the short interval of time, the option value does not expire when the project expires [so that lambda in Equation (7) always equals 0], and the option value is not a function of time [so that the derivative of $F(P)$ with respect to t in Equation (8) is zero].

[35] See Avinash K. Dixit and Robert S. Pindyck, *Investment Under Uncertainty*, New Jersey: Princeton University Press, 1994 for a discussion of these two terms.